

## 3.2

# Planetary and Satellite Motion

A perfectly executed football or hockey pass is an amazing achievement. A fast-moving player launches an object toward the place where a fast-moving receiver will most likely be when the ball or puck arrives. The direction and force of the pass are guided by intuition and skill acquired from long practice.

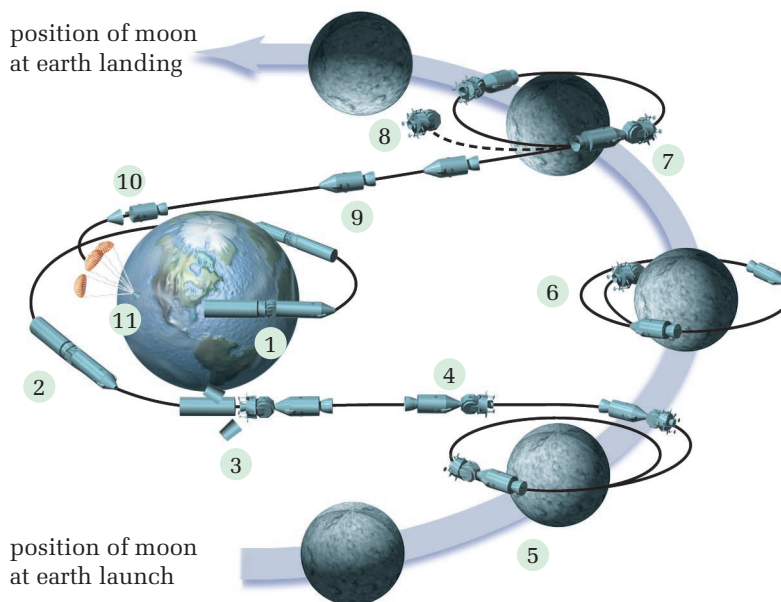
Launching a spacecraft has the same objective, but extreme precision is required — the outcome is more critical than that of an incomplete pass. Scientists and engineers calculate every detail of the trajectories and orbits in advance. The magnitude and direction of the forces and the time interval for firing the rockets are analyzed and specified in minute detail. Even last-minute adjustments are calculated exhaustively. The process can be extremely complex, but it is based on principles that you have already studied — the dynamics of circular motion and the law of universal gravitation.

### SECTION EXPECTATION

- Use Newton's law of universal gravitation to explain planetary and satellite motion.

### KEY TERMS

- geostationary orbit
- microgravity
- perturbation



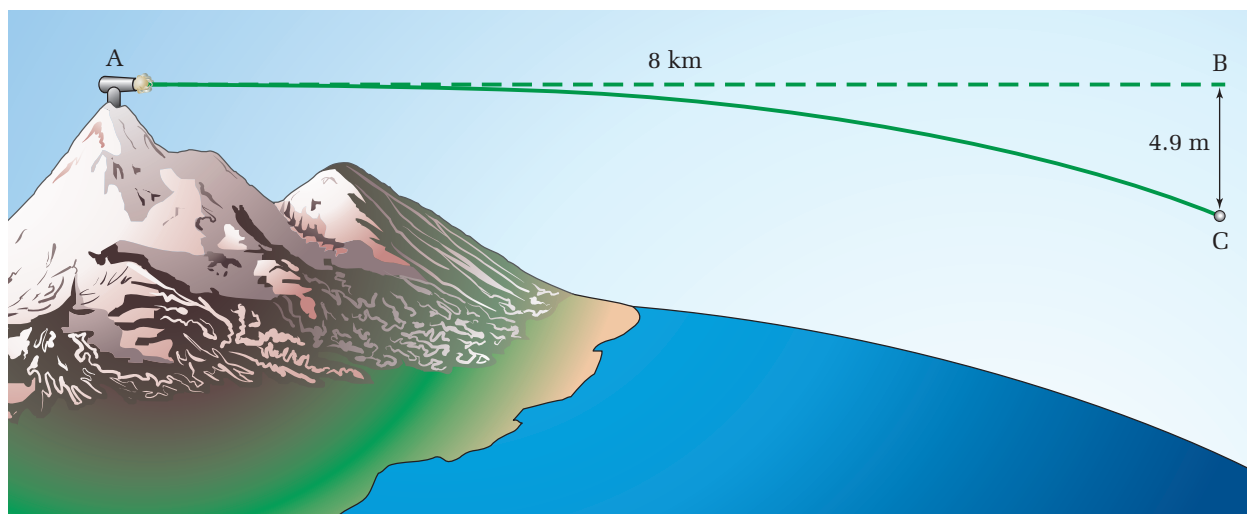
**Figure 3.5** The steps in a typical *Apollo* lunar mission are: (1) lift off and enter Earth orbit, (2) leave Earth orbit, (3) release booster rocket, turn, and dock with lunar module that is stored between the booster rocket and the service module, (4) make a mid-course correction, (5) enter lunar orbit, (6) command module continues to orbit the Moon, while the lunar module descends to the lunar surface, carries out tasks, ascends, and reconnects with the command module, (7) leave lunar orbit, (8) eject lunar module, (9) mid-course correction, (10) eject service module, and (11) command module lands in the Pacific Ocean.

## Newton's Mountain

The planets in our solar system appear to have been “orbiting” the Sun while they were forming. Great swirling dust clouds in space began to condense around a newly formed Sun until they finally became the planets. How, then, do artificial satellites begin orbiting Earth?

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 3.6. The cannon ball would certainly fall toward Earth. If the cannon ball travelled far enough horizontally while it fell, however, the curvature of Earth would be such that Earth's surface would “fall away” as fast as the cannon ball fell.

You can determine how far the cannon ball will fall in one second by using the kinematic equation  $\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$ . If a cannon ball had zero vertical velocity at time zero, in one second it would fall a distance  $\Delta d = 0 + \frac{1}{2} \left( -9.81 \frac{\text{m}}{\text{s}^2} \right) (1 \text{ s})^2 = -4.9 \text{ m}$ . From the size and curvature of Earth, Newton knew that Earth's surface would drop by 4.9 m over a horizontal distance of 8 km.



**Figure 3.6** The values shown here represent the distance that the cannon ball would have to go in one second in order to go into orbit.

Newton's reasoning was absolutely correct, but he did not account for air friction. Although the air is too thin to breathe easily on top of Mount Everest, Earth's highest mountain, it would still exert a large amount of air friction on an object moving at 8 km/s. If you could take the cannon to 150 km above Earth's surface, the atmosphere would be so thin that air friction would be negligible. Newton understood how to put an artificial satellite into orbit, but he did not have the technology.

Today, launching satellites into orbit is almost routine, but the scientists and engineers must still carefully select an orbit and perform detailed calculations to ensure that the orbit will fulfil the purpose of the satellite. For example, some weather satellites orbit over the Poles at a relatively low altitude in order to collect data in detail. Since a satellite is constantly moving in relation to a ground observer, the satellite receiver has to track the satellite continually so that it can capture the signals that the satellite is sending. In addition, the satellite is on the opposite side of Earth for long periods of time, so several receivers must be located around the globe to collect data at all times.

Communication satellites and some weather satellites travel in a **geostationary orbit** over the equator, which means that they appear to hover over one spot on Earth's surface at all times. Consequently, a receiver can be aimed in the same direction at all times and constantly receive a signal from the satellite. The following problem will help you to find out how these types of orbits are attained.

Arthur C. Clarke (1917–), scientist and science fiction writer, wrote a technical paper in 1945, setting out the principles of geostationary satellites for communications. Many scientists in the field at the time did not believe that it was possible. Today, geostationary orbits are sometimes called “Clarke orbits.” Clarke also co-authored the book and movie *2001: A Space Odyssey*.

## SAMPLE PROBLEM

### Geostationary Orbits

**At what velocity and altitude must a satellite orbit in order to be geostationary?**

#### Conceptualize the Problem

- A satellite in a *geostationary orbit* must remain over the *same point* on Earth at all times.
- To be geostationary, the satellite must make *one complete orbit* in exactly the same time that Earth rotates on its axis. Therefore, the period must be *24 h*.
- The *period* is related to the *velocity* of the satellite.
- The *velocity* and *altitude* of the satellite are determined by the amount of *centripetal force* that is causing the satellite to remain on a circular path.
- Earth's *gravity* provides the *centripetal force* for satellite motion.
- The values for the mass and radius of Earth are listed in Appendix B, Physical Constants and Data.

#### Identify the Goal

- (a) The velocity,  $v$ , of a geostationary satellite
- (b) The altitude,  $h$ , of a geostationary satellite

continued ►

## Identify the Variables and Constants

### Known

Orbit is geostationary.

### Implied

$$T = 24 \text{ h}$$

$$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$m_E = 5.98 \times 10^{24} \text{ kg}$$

$$r_E = 6.38 \times 10^6 \text{ m}$$

### Unknown

 $r$  (radius of satellite's orbit) $h$ 

## Develop a Strategy

To find the velocity, start by setting the gravitational force equal to the centripetal force.

Simplify the expression.

$$F_g = F_c$$

$$G \frac{m_E m_s}{r^2} = \frac{m_s v^2}{r}$$

$$G \frac{m_E}{r} = v^2$$

To eliminate  $r$  from the equation, use the equation for the definition of velocity and solve for  $r$ . Recall that the period,  $T$ , is known.

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r$$

$$\Delta t = T$$

$$v = \frac{2\pi r}{T}$$

$$r = \frac{vT}{2\pi}$$

Substitute the expression for  $r$  into the equation for  $v$  above and solve for  $v$ .

$$G \frac{m_E}{\frac{vT}{2\pi}} = v^2$$

$$v^2 = \frac{2\pi G m_E}{vT}$$

$$v^3 = \frac{2\pi G m_E}{T}$$

$$v = \sqrt[3]{\frac{2\pi G m_E}{T}}$$

Convert  $T$  to SI units.

$$T = (24 \text{ h}) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 8.64 \times 10^4 \text{ s}$$

Substitute numerical values and solve.

$$v = \sqrt[3]{\frac{2\pi \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) (5.98 \times 10^{24} \text{ kg})}{8.64 \times 10^4 \text{ s}}}$$

$$v = \sqrt[3]{2.9006 \times 10^{10} \frac{\text{m}^3}{\text{s}^3}}$$

$$v = 3.0724 \times 10^3 \frac{\text{m}}{\text{s}}$$

$$v \cong 3.07 \times 10^3 \frac{\text{m}}{\text{s}}$$

- (a) The orbital velocity of the satellite is about  $3.07 \times 10^3 \text{ m/s}$ , which is approximately 11 000 km/h.

To find the altitude of the satellite, substitute the value for velocity into the expression above that you developed to find  $r$  in terms of  $v$ .

$$r = \frac{vT}{2\pi}$$

$$r = \frac{\left(3.0724 \times 10^3 \frac{\text{m}}{\text{s}}\right)(8.64 \times 10^4 \text{ s})}{2\pi}$$

$$r = 4.2249 \times 10^7 \text{ m}$$

The calculated value for  $r$  is the distance from Earth's centre to the satellite. To find the altitude of the satellite, you must subtract Earth's radius from  $r$ .

$$r = r_E + h$$

$$h = r - r_E$$

$$h = 4.2249 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m}$$

$$h = 3.5869 \times 10^7 \text{ m}$$

$$h \cong 3.59 \times 10^7 \text{ m}$$

- (b) The altitude of all geostationary satellites must be  $3.59 \times 10^7 \text{ m}$ , or  $3.59 \times 10^4 \text{ km}$ , above Earth's surface.

### Validate the Solution

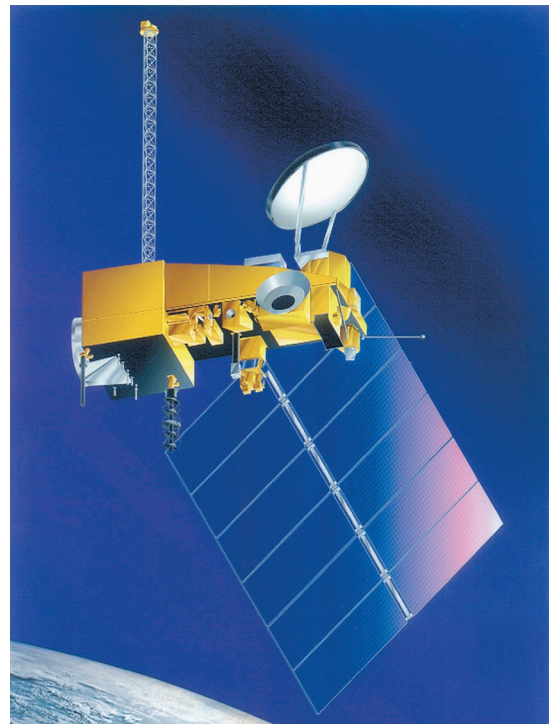
A velocity of 11 000 km/h seems extremely fast to us, but the satellite is circling Earth once per day, so the velocity is reasonable.

Check the units for velocity.  $\sqrt[3]{\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \text{kg}} = \sqrt[3]{\frac{\text{N} \cdot \text{m}^2 \cdot \text{kg}}{\text{s} \cdot \text{kg}^2}} = \sqrt[3]{\frac{\text{kg} \cdot \text{m}^3}{\text{s} \cdot \text{kg}}} = \sqrt[3]{\frac{\text{m}^3}{\text{s}^3}} = \frac{\text{m}}{\text{s}}$

### PRACTICE PROBLEMS

15. The polar-orbiting environmental satellites (POES) and some military satellites orbit at a much lower level in order to obtain more detailed information. POES complete an Earth orbit 14.1 times per day. What are the orbital speed and the altitude of POES?
16. The International Space Station orbits at an altitude of approximately 226 km. What is its orbital speed and period?
17. (a) The planet Neptune has an orbital radius around the Sun of about  $4.50 \times 10^{12} \text{ m}$ . What are its period and its orbital speed?
- (b) Neptune was discovered in 1846. How many orbits has it completed since its discovery?

NASA operates two polar-orbiting environmental satellites (POES) designed to collect global data on cloud cover; surface conditions such as ice, snow, and vegetation; atmospheric temperatures; and moisture, aerosol, and ozone distributions.







## “Weightlessness” in Orbit

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 3.7. The astronauts appear to float freely in the spacecraft and they describe the condition as being “weightless.” Is a 65 kg astronaut in a space station weightless? Check it out, using the following calculation.

**Figure 3.7** If an astronaut in a spacecraft dropped an apple, it would fall toward Earth, but it would not look as though it was falling.

- The space station orbits at an altitude of approximately 226 km. Find the radius of its orbit by adding the altitude to Earth’s radius.
- Use Newton’s law of universal gravitation to find the astronaut’s weight.

$$r = r_E + h$$

$$r = 6.38 \times 10^6 \text{ m} + 226 \text{ km} \left( \frac{1000 \text{ m}}{\text{km}} \right)$$

$$r = 6.606 \times 10^6 \text{ m}$$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$F_g = \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(5.98 \times 10^{24} \text{ kg})(65 \text{ kg})}{(6.606 \times 10^6 \text{ m})^2}$$

$$F_g = 594 \text{ N}$$

The astronaut’s weight on Earth would be

$F_g = mg = (65 \text{ kg})(9.81 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}) \cong 638 \text{ N}$ . There is very little difference between the astronaut’s weight on Earth and in the space station. Why do astronauts appear “weightless” in space? Think back to your calculations of apparent weight in Chapter 1, Fundamentals of Dynamics. When the imaginary elevator that you were riding in was falling with an acceleration of  $9.81 \text{ m/s}^2$ , the scale you were standing on read zero newtons. Your apparent weight was zero because you, the scale, and the elevator were falling with the same acceleration. You and the scale were not exerting any force on each other.

The same situation exists in the space station and all orbiting spacecraft all of the time. As in the case of Newton’s cannon ball, everything falls to Earth with the same acceleration, but Earth is “falling away” equally as fast. You could say that a satellite is “falling around Earth.” Some physicists object to the term “weightlessness” because, as you saw, there is no such condition. NASA coined the term **microgravity** to describe the condition of apparent weightlessness.

### WEB LINK

[www.mcgrawhill.ca/links/physics12](http://www.mcgrawhill.ca/links/physics12)

To learn about the effects of weightlessness on the human body, go to the above Internet site and click on **Web Links**.